Calculating the Value of an Annuity.
<table>
<thead>
<tr>
<th>Time Value of Money</th>
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- **Time value of money is used to find the value of**
  - Single amounts
  - Annuities

- **Effects of multiple compounding periods**

Time value of money is used to find the value of single amounts and annuities. This example will show the effects of multiple compounding periods on amounts of money.
## Calculating Present and Future Values

We can use:
- Algebra.
- Time value tables (avoids the use of algebra, but limited usefulness).
- Financial calculator.
- Functions on spreadsheets.

In calculating the time value of money equations, we can use algebra, time value tables, financial calculator, and functions on spreadsheets.
An annuity is a cash flow of equal sums (typically annual).

**Types of annuities:**
- **Ordinary annuity:** cash flows occur at the end of each annuity period
- **Annuity due:** cash flows occur at the beginning of each annuity period

An annuity is a cash flow of equal sums.

There are two types of annuities: an **ordinary annuity**, which has a cash flow occur at the end of the period, and an **annuity due**, which has the cash flow occur at the beginning of the period.
Future Value - Ordinary Annuity

$100 per year for 3 years at 10%.

- **Algebraic solution**

\[
FVA_n = PMT \left( \frac{(1+i)^n - 1}{i} \right)
\]

\[
= 100 \left( \frac{(1+0.10)^3 - 1}{0.10} \right) = 331.00
\]

- **Variation with time value tables**

\[
FVA_n = PMT \left( FVIFA_{i,n} \right)
\]

\[
= 100 \left( 3.310 \right) = 331.00
\]

In this example we will look at the future value if we receive an equal series of payments using compound interest. The algebraic solution demonstrates how complex it can be to find the value. Using the time value tables, is accomplished by taking the payment amount and multiplying that by the future value interest factor of an annuity that corresponds with the three periods at 10%.
Future Value of an Annuity Due

- **Algebraic solution**

\[
FVA_n = PMT \left( \frac{(1+i)^n - 1}{i} \right) (1+i)
\]

\[
= \$100 \left( \frac{(1 + 0.10)^3 - 1}{0.10} \right) (1.10) = \$364.10
\]

- **Variation with time value tables**

\[
FVA_n = PMT \left( FVIFA_{i,n} \right) (1+i)
\]

\[
= \$100 \left( 3.310 \right) (1.10) = \$364.10
\]

With the future value of an annuity due, we are looking for the future value of a series of equal annuity payments made or received at the beginning of each period. If solving using the algebraic solution, it would look like this. If using the time value tables, the future value interest factor is multiplied by one plus the interest rate (or 10%) and then multiplied by the payment amount of $100.
$100 per year for 3 years at 10% 

\[ PVA_n = PMT \left( \frac{1}{i} \frac{1}{(1+i)^n} \right) = 100 \left( \frac{1}{0.10} \frac{1}{(1+0.10)^3} \right) = 248.69 \]

**Variation with time value tables**

\[ PVA_n = PMT \left( PVIFA_{i,n} \right) = 100 \left( 2.4869 \right) = 248.69 \]

In this example we will look at the present value of an ordinary annuity or the amount of a series of equal payments in the future is worth today taking into account the time value of money. The algebraic solution demonstrates how complex it can be to find the value. Using the time value tables is accomplished by taking the payment amount and multiplying that by the present value interest factor of an annuity that corresponds with three periods at 10%.
## Present Value of an Annuity Due

### Algebraic solution

\[
PVA_n = PMT \left( \frac{1 - \frac{1}{(1 + i)^n}}{i} \right) (1 + i) = $100 \left( \frac{1 - \frac{1}{(1 + 0.10)^3}}{0.10} \right) (1.10) = $273.55
\]

### Variation with time value tables

\[
PVA_n = PMT \left( PVIFA_{i,n} \right) (1 + i)
= $100 \left( 2.4869 \right) (1.10) = $273.55
\]

With the present value of an annuity due, we are looking for the present value for a series of equal annuity payments made or received at the beginning of each period. If solving using the algebraic solution, it would look like this. If using the time value tables, the present value interest factor is multiplied by one plus the interest rate (or 10%) and then multiplied by the payment amount of $100.